

# Prediction of soil thermal conductivity based on penetration resistance and water content or air-filled porosity

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## Abstract

Accurate data of thermal conductivity are required in many agricultural, meteorological and engineering applications. New regression equations for predicting thermal conductivity based on easily measured quantities such as penetration resistance and water content or air-filled porosity are presented. The thermal conductivities from the equations are compared with those from a statistical-physical model of a good estimation capability. The measurements of the quantities were done on silt loam in a sloping vineyard (Italy) at various times and locations to get a wide range of measured values. It is shown that the performance of the equations relating the thermal conductivity with penetration resistance and air-filled porosity is greater ( $R^2 = 0.94$ ) than with penetration resistance and volumetric water content ( $R^2 = 0.77$ ). Therefore, the equations based on measured penetration resistance and air-filled porosity are recommended for predicting the thermal conductivity of the soil. Adding sand content and transformation of strength values to root squares somewhat improved the predictions. To minimize the effects of spatial variability of the measured quantities on the thermal conductivity and to reduce measurement time and soil disturbance, systems for combined measurements of penetration resistance and water content at the same place need to be used in further studies.

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## 1. Introduction

Soil thermal properties, including thermal conductivity, heat capacity, and thermal diffusivity, play an important role in the surface-energy partitioning and resulting temperature distribution [8]. Among the thermal properties the thermal conductivity is most widely used in numerous meteorological and engineering applications, e.g. [7]. Consistent estimates of the thermal conductivity in extraterrestrial soil analogues are useful in understanding the nature of thermal evolution under space conditions, e.g. [14,27]. The thermal conductivity of soil influences the microclimate for plant growth [12].

The thermal conductivity can be determined based on measurements of temperature in response to heat input

[15], or by a heat-pulse probe [5]. More recently, multi-needle heat-pulse probes have been developed [3,24]. The probe developed by Bristow et al. [3] allows for simultaneous measurement of soil thermal properties, water content and electrical conductivity, and is particularly suitable for use near the soil surface or near the roots because of its small size. Wide use of the measurement methods in situ is still limited by time-consuming measurement procedures and relatively high cost of measuring instruments.

The above drawbacks imply a need for the development of other approaches that could determine the thermal conductivity based on easily measured quantities. It is well known that water content and bulk density significantly influence both thermal conductivity, e.g. [1,2,10,17,29], and penetration resistance [4,19]. Since the penetration resistance is more easily and cheaply measured than the thermal conductivity, attempts are undertaken to predict the thermal conductivity based on the penetration resistance in combination with other quantities. Research under

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## Nomenclature

$a$	parameter of equation associated with PR or root square of PR	<i>Greek symbols</i>	
$b$	parameter of equation associated with $\theta$ or $\theta_s - \theta$	$\theta$	water content ( $\text{m}^3 \text{m}^{-3}$ )
$b_1$	parameter of equation associated with $S$	$\theta_s$	saturated water content ( $\text{m}^3 \text{m}^{-3}$ )
$B$	regression coefficients $a, b, b_1$ and $c$	$\theta_s - \theta$	air-filled porosity ( $\text{m}^3 \text{m}^{-3}$ )
$c$	intercept	$\phi$	porosity ( $\text{m}^3 \text{m}^{-3}$ )
$f_w$	content of liquid ( $\text{m}^3 \text{m}^{-3}$ )	$\lambda$	thermal conductivity, predicted ( $\text{W m}^{-1} \text{K}^{-1}$ )
$f_g$	content of air ( $\text{m}^3 \text{m}^{-3}$ )	$\lambda_m$	thermal conductivity, measured ( $\text{W m}^{-1} \text{K}^{-1}$ )
$f_s$	content of minerals and organic matter ( $\text{m}^3 \text{m}^{-3}$ )	$\lambda_o$	thermal conductivity, “observed” ( $\text{W m}^{-1} \text{K}^{-1}$ )
$L$	number of all combinations of particles	$\lambda_1, \lambda_2, \dots, \lambda_k$	thermal conductivity of particles ( $\text{W m}^{-1} \text{K}^{-1}$ )
MRE	maximum relative error (%)	$\lambda_q$	thermal conductivity of quartz
PR	penetration resistance (MPa)	$\lambda_{mi}$	thermal conductivity of other minerals
$r_1, r_2, \dots, r_k$	radii of particles (m)	$\lambda_{om}$	thermal conductivity of organic matter
$R^2$	determination coefficient	$\lambda_w$	thermal conductivity of water
RMSE	root mean square error ( $\text{W m}^{-1} \text{K}^{-1}$ )	$\lambda_g$	thermal conductivity of air
$S$	content of sand (2–0.02 mm) ( $\text{m}^3 \text{m}^{-3}$ )	<i>Subscripts</i>	
$T$	temperature ( $^\circ\text{C}$ )	w	water
$u$	number of parallel connections of thermal resistors	g	air
$x_1, x_2, \dots, x_k$	number of particles	s	solid

controlled space conditions indicated a positive relationship between the penetration resistance and thermal conductivity [21,26]. However, according to our knowledge, these relationships were not studied using data of penetration resistance from field measurements. Some recent studies revealed that the thermal conductivity is more strongly correlated with air-filled porosity than with volume fractions of water or solids [23]. Also the thermal conductivity of porous ice was influenced by the volume of air and pore size and tortuosity [25].

Therefore, the objective of the study was to determine the relationships between the soil thermal conductivity and penetration resistance in combination with water content or air-filled porosity.

## 2. Materials and methods

### 2.1. Soil and measurements of soil properties

The study was conducted on silt loam (*Eutrochrepts*) in a sloping vineyard (Piedmontese hillside, N-W Italy). The research area included four plots of 30 m long and 2.7 m wide. The measurements of penetration resistance, bulk density and water content were performed on four transects in each plot. Penetration resistance was measured with a recording penetrometer with a cone angle of  $30^\circ$  and  $1 \text{ cm}^2$  area [38] at intervals of 2.5 cm down the soil profile to a depth of 25 cm. The number of penetration resistance measurements was 1440 (3 penetrations  $\times$  3 measuring points  $\times$  10 depth intervals  $\times$  4 transects  $\times$  4 plots).

At the same time,  $100 \text{ cm}^3$  cores and bulk soil samples were taken at 2.5–7.5, 10–15 and 17.5–22.5 cm depths to determine bulk density, gravimetric water content and content of sand, silt, clay and organic matter using standard methods. The sampling was from places close to measuring points of penetration resistance to minimize the effects of spatial variability. Volumetric water content was calculated on the basis of the gravimetric water content and bulk density. Air-filled porosity ( $\theta_s - \theta$ ) was obtained from the difference between volumetric water contents at saturation ( $\theta_s$ ), determined in laboratory, and at current state ( $\theta$ ). The number of measurements was 144 (3 replicates  $\times$  3 depths  $\times$  4 transects  $\times$  4 plots). The data from the cores and bulk soil were referred to penetration resistance measurements at 0–7.5, 7.5–15 and 15–25 cm depths, respectively.

The results of water content, bulk density, organic matter, content of quartz and other minerals (as determined based on content of sand, silt and clay) were used to estimate the thermal conductivity using the statistical-physical model [30] and those of penetration resistance, contents of water and sand to predict the thermal conductivity using non-linear regression equations.

### 2.2. Statistical-physical model

Since proper measurements of the thermal conductivity under changeable meteorological conditions in the field are difficult and the obtained results are often uncertain, we conducted a laboratory study under controlled conditions

to get reliable data of thermal conductivity at various water content and bulk density of the silty loam soil. The results obtained were used for verification of the statistical-physical model that predicts the thermal conductivity based on soil mineralogical composition, water content, and bulk density. Such verification using measured rather than adjustable parameters is a convenient feature of the model.

The thermal conductivity of soil,  $\lambda_o$  ( $\text{W m}^{-1} \text{K}^{-1}$ ), was calculated using the physical-statistical model [30,34] as described by the following equations:

$$\lambda_o = \frac{4\pi}{u \sum_{j=1}^L \frac{P(x_{1j}, \dots, x_{kj})}{x_{1j}\lambda_1(T)r_1 + \dots + x_{kj}\lambda_k(T)r_k}} \quad (1)$$

where  $u$  is the number of parallel connections of soil particles treated as thermal resistors,  $L$  is the number of all possible combinations of particle configuration,  $x_1, x_2, \dots, x_k$  – a number of individual particles of a soil with thermal conductivity  $\lambda_1, \lambda_2, \dots, \lambda_k$  and particle radii  $r_1, r_2, \dots, r_k$ , where  $\sum_{i=1}^k x_{ij} = u$ ,  $j = 1, 2, \dots, L$ ,  $P(x_{ij})$  – probability of occurrence of a given soil particle configuration calculated from the polynomial distribution [11]:

$$P(x_{1j}, \dots, x_{kj}) = \frac{u!}{x_{1j}! \dots x_{kj}!} f_1^{x_{1j}} \dots f_k^{x_{kj}} \quad (2)$$

The condition:  $\sum_{j=1}^L P(X = x_j) = 1$  must also be fulfilled. The probability of selecting a given soil constituent (particle)  $f_i$ ,  $i = s, w, g$ , in a single trial was determined based on fundamental physical soil properties. In this case,  $f_s$ ,  $f_w$ , and  $f_g$  are the content of individual minerals and organic matter –  $f_s = 1 - \phi$ , water –  $f_w = \theta$  and air –  $f_g = \phi - \theta$  in a unit of volume,  $\phi$  – soil porosity. The data on texture, organic matter content and solid phase densities of soil and organic matter were used to determine the probability of occurrence of a given soil component. It was assumed that sand fraction consists mainly of quartz; however, other minerals are contained in a majority of silt and clay fractions [10]. Based on the soil textural composition and solid phase density, the content of quartz and other minerals and organic matter per unit volume was calculated.

So far the investigations showed that to calculate the soil thermal conductivity the conductivities of the main soil components could be used [30]. They are: quartz, other minerals, organic matter, water and air. Their values of thermal conductivity and relations to temperature are presented in Table 1.

Table 1  
Expressions and values for components used in calculating the thermal conductivity of soils ( $T$  in  $^{\circ}\text{C}$ )

Source	Components	Expression, values ( $\text{W m}^{-1} \text{K}^{-1}$ )
	quartz – $\lambda_q$ ,	$9.103 - 0.028T$
De Vries [10]	other minerals – $\lambda_{mi}$ ,	2.93
De Vries [10]	organic matter – $\lambda_{om}$ ,	0.251
Kimball et al. [16]	water – $\lambda_w$ ,	$0.552 + 2.34 \times 10^{-3}T - 1.1 \times 10^{-5}T^2$
Kimball et al. [16]	air – $\lambda_g$ ,	$0.0237 + 0.000064T$

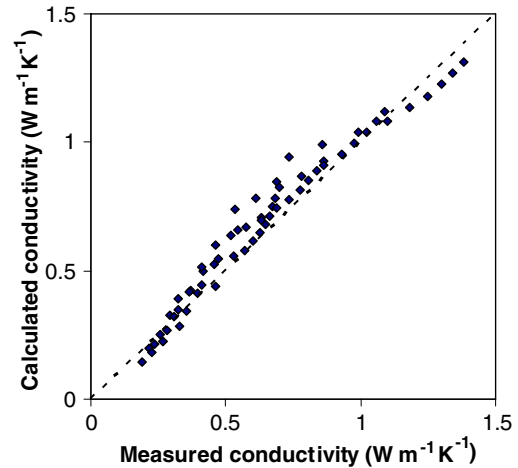


Fig. 1. Comparison of  $\lambda$  measured and estimated by means of the statistical-physical model for silt loam soil. Dotted lines represent the relation 1:1. Equations of linear regression are:  $\lambda_o = 1.0279\lambda_m + 0.0338$ ,  $\text{RMSE} = 0.093 \text{ W m}^{-1} \text{K}^{-1}$ ,  $\text{MRE} = 38.3 \%$ ,  $R^2 = 0.953$ .

It was shown that predictions of  $\lambda$  by means of the model, for a wide range of soil types at various water content, bulk density and temperature ( $T$ ), were in good agreement with the measured values [31,35] and with those of the widely used De Vries model [10]. Fig. 1 illustrates an example of a good agreement between measured [18] and estimated  $\lambda$  for the studied silt loam soil [30,31,35]. Taking into consideration good performance of the statistical-physical model we used its output thermal conductivities as reference (dependent variable) for the conductivities derived from regression equations developed in the present study. In subsequent parts of the paper the reference data will be called “observed data”.

### 2.3. Regression equations

The fixed non-linear regression procedure in the program Statistica 6 [28] was used to develop the regression equations to relate the observed thermal conductivity to the measured soil penetration resistance, water content and content of sand. We selected this procedure based on earlier results indicating non-linear relation between the  $\lambda$  and some quantities of porous media [9]. The data of the independent variables were transformed in the procedure with different non-linear functions including:  $x^2 \dots x^5$ ,  $\sqrt{x}$ ,  $\ln x$ ,  $\log x$ ,  $e^x$ ,  $10^x$ ,  $\frac{1}{x}$ .

Beta coefficients being the regression coefficients were derived from standardization of variables to a mean of zero and a standard deviation of one to compare the relative contribution of each independent variable in the prediction of the thermal conductivity.  $B$  coefficients that are not standardized were used to calculate the thermal conductivity from the equations derived with the procedure of fixed non-linear regression.

Eight regression Eqs. (3)–(10) providing the most accurate predictions for the thermal conductivity were selected.

The equations are based on penetration resistance and water content or penetration resistance and air-filled porosity (difference of saturated and current volumetric water content). Moreover, content of sand was included because of its significant effect on the thermal conductivity and spatial variability in the field [32,33]:

$$\lambda = a \cdot PR + b \cdot \theta + c \tag{3}$$

$$\lambda = a \cdot PR + b \cdot \theta + b_1 S + c \tag{4}$$

$$\lambda = a \cdot \sqrt{PR} + b \cdot \theta + c \tag{5}$$

$$\lambda = a \cdot \sqrt{PR} + b \cdot \theta + b_1 S + c \tag{6}$$

$$\lambda = a \cdot PR + b \cdot (\theta_s - \theta) + c \tag{7}$$

$$\lambda = a \cdot PR + b \cdot (\theta_s - \theta) + b_1 S + c \tag{8}$$

$$\lambda = a \cdot \sqrt{PR} + b \cdot (\theta_s - \theta) + c \tag{9}$$

$$\lambda = a \cdot \sqrt{PR} + b \cdot (\theta_s - \theta) + b_1 S + c \tag{10}$$

where  $\lambda$  ( $W m^{-1} K^{-1}$ ) is the thermal conductivity;  $a, b, b_1, c$  are the parameters that are denoted as  $B$  coefficients; PR (MPa) is the penetration resistance;  $\theta$  ( $m^3 m^{-3}$ ) is the current soil water content;  $\theta_s$  ( $m^3 m^{-3}$ ) is the soil water content at saturation;  $S$  ( $m^3 m^{-3}$ ) is the sand (2–0.02 mm) content.

#### 2.4. Statistical evaluation

The error level understood as the difference between the data predicted by the regression equations and observed was evaluated on the basis of the root mean square error (RMSE), the maximum relative error (MRE), the determination coefficients ( $R^2$ ) and the linear regression coefficients. The root mean square error was calculated as

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (f_{mi} - f_{ci})^2}{k}} \tag{11}$$

where,  $f_{mi}$  is the measured value,  $f_{ci}$  is the calculated value,  $k = n - 1$  if  $n < 30$  and  $k = n$  if  $n > 30$ ,  $n$  is the number of data. The maximum relative error was calculated from the equation:

$$MRE = \max_{i=1,2,\dots,n} \left\{ \left| \frac{f_{mi} - f_{ci}}{f_{mi}} \right| \cdot 100\% \right\} \tag{12}$$

Lesser values of RMSE and MRE indicate better agreement between the values estimated and measured, or lesser deviation.

### 3. Results and discussion

The values of the measured soil penetration resistance, water content, bulk density and sand (2–0.02 mm) as independent variables varied in the ranges of  $0.1 \div 9.3$  MPa;  $0.096 \div 0.474$   $m^3 m^{-3}$ ,  $0.96 \div 1.48$   $mg m^{-3}$ , and  $298.9 \div 347.4$   $g kg^{-1}$ , respectively. The ranges for content of silt (0.02–0.002 mm), clay (<0.002 mm) and organic matter content were  $558.4 \div 592.2$   $g kg^{-1}$ ,  $94.2 \div 113.6$   $g kg^{-1}$ ,  $26.8 \div 78.0$   $g kg^{-1}$ , respectively.

The observed values of the thermal conductivity were regressed against penetration resistance and water content or air-filled porosity and sand content. Parameters of the resulting equations are given in Table 2. Other statistical parameters for all the regression equations are given in Tables 3 and 4. Only Eqs. (5) and (9) were selected as examples for graphical presentation and more detailed interpretation because they are based on directly measured penetration resistance and water content and predict well the thermal conductivity.

Statistical analysis showed a significant effect of both water content and penetration resistance on the thermal conductivity in Eq. (5) (Tables 2–4). Values of  $beta$  coefficients (Table 2) demonstrate that the relative contribution of water (0.924) in the prediction of the thermal conductivity is substantially greater than that of penetration resistance (0.195).  $B$  coefficients, associated with  $beta$  coefficients, were used in the regression equations to predict the thermal conductivities. The coefficients represent regression coefficients  $a, b, b_1$  and  $c$  in a given equation;  $a$  is associated with PR or root square of PR,  $b$  – with  $\theta$  (water content) or  $\theta_s - \theta$  (air-filled porosity),  $b_1$  – with  $S$  (sand content), and  $c$  is the intercept.

Table 2  
Relative  $Beta$  and  $B$  regression coefficients for different combinations of independent variables in Eqs. (3)–(10)

Statistics	$Beta$	$B$	$Beta$	$B$	$Beta$	$B$	$Beta$	$B$
	Eq. (3)		Eq. (4)		Eq. (5)		Eq. (6)	
Intercept		0.156		−0.077		0.065		−0.222
PR (MPa)	0.193	0.034	0.216	0.038	0.195	0.117	0.223	0.134
$\theta$ ( $m^3 m^{-3}$ )	0.926	2.674	0.935	2.699	0.924	2.668	0.935	2.698
$S$ ( $m^3 m^{-3}$ )	–	–	0.043	0.007	–	–	0.051	0.008
	Eq. (7)		Eq. (8)		Eq. (9)		Eq. (10)	
Intercept		1.526		1.277		1.471		1.187
PR (MPa)	0.113	0.020	0.138	0.024	0.114	0.068	0.141	0.008
$\theta_s - \theta$ ( $m^3 m^{-3}$ )	−0.993	−2.392	−1.002	−2.414	−0.991	−2.388	−1.001	−2.412
$S$ ( $m^3 m^{-3}$ )	–	–	0.048	0.008	–	–	0.053	0.085

$B$  ( $a \leftrightarrow PR$ ;  $b \leftrightarrow \theta$ ;  $b_1 \leftrightarrow S$ ;  $c \leftrightarrow$  intercept)  $\theta$  – water content;  $\theta_s$  – saturated water content;  $S$  – content of sand; PR – penetration resistance;  $\leftrightarrow$  – means parameter corresponding to a given variable.

Table 3

Summary statistics of comparison of observed and predicted thermal conductivities for different combinations of independent variables in Eqs. (3)–(10)

Statistics	Eq. (3)	Eq. (4)	Eq. (5)	Eq. (6)	Eq. (7)	Eq. (8)	Eq. (9)	Eq. (10)
RMSE	0.135	0.134	0.134	0.134	0.068	0.067	0.067	0.066
MRE	61.9	60.5	60.0	60.0	35.3	33.7	34.2	32.3
$R^2$	0.767	0.769	0.768	0.770	0.941	0.943	0.942	0.944

RMSE ( $\text{W m}^{-1} \text{K}^{-1}$ ) – root mean square error, MRE (%) – maximum relative error,  $R^2$  – determination coefficient.

Table 4

Basic statistics of observed, predicted and residuals of the thermal conductivity for different combinations of independent variables in Eqs. (3)–(10)

Statistics	Observed	Predicted	Residual	Predicted	Residual	Predicted	Residual	Predicted	Residual
		Eq. (3)		Eq. (4)		Eq. (5)		Eq. (6)	
Minimum	0.400	0.430	−0.371	0.411	−0.362	0.404	−0.375	0.378	−0.369
Maximum	1.529	1.502	0.417	1.525	0.432	1.508	0.412	1.535	0.424
Mean	0.984	0.984	0.000	0.984	0.000	0.984	0.000	0.984	0.000
Median	1.006	1.007	−0.001	1.008	−0.002	1.005	0.003	1.006	0.001
		Eq. (7)		Eq. (8)		Eq. (9)		Eq. (10)	
Minimum	0.400	0.454	−0.217	0.462	−0.210	0.444	−0.219	0.451	−0.214
Maximum	1.529	1.555	0.183	1.549	0.198	1.558	0.179	1.552	0.193
Mean	0.984	0.984	0.000	0.984	0.000	0.984	0.000	0.984	0.000
Median	1.006	0.958	0.007	0.964	0.008	0.959	0.005	0.963	0.006

Comparison of the observed and predicted thermal conductivity (Fig. 2a) indicates a considerable dispersion of the values with the residuals varying from  $-0.375$  to  $+0.412$  with mean of zero (Table 4). The data in Table 3 indicate that penetration resistance and volumetric water content accounted for 77% of the variation in thermal conductivity with RMSE of  $0.134 \text{ W m}^{-1} \text{ K}^{-1}$  and MRE of 60%. To determine the effect of the independent variables on the thermal conductivity and deformation in the distribution of the predicted  $\lambda$  we compared the distributions of observed and predicted data with consideration of the statistical parameters. The observed thermal conductivity varied from  $0.4$  to  $1.529 \text{ W m}^{-1} \text{ K}^{-1}$  with the maximum of about 200 observations with  $\lambda$  value of approximately  $1 \text{ W m}^{-1} \text{ K}^{-1}$ . As can be seen from Fig. 2b the distribution of the observed data is close to the theoretical normal distribution (solid line) with a small left side asymmetry (skewness =  $-0.19$ ) and small flattening (kurtosis =  $-0.8$ ) as compared to the normal distribution. The distribution for the predicted  $\lambda$  using Eq. (5) (Fig. 2c) is somewhat different than that for observed data (Fig. 2b). The former has a slight deformation as shown by clear drop of number of data corresponding to the maximum value of the observed data that can be due to that the real non-linearity of the independent variables was different than non-linearity of the function used in the present study. As a consequence the function does not predict satisfactorily the effect of PR and water content on  $\lambda$ .

Comparison of histogram of residuals and normal probability plot of residuals with theoretical normal distribution (solid line) (Fig. 2d and e), as well as similar mean and median values (Table 4), indicate a good agreement of the residuals with the theoretical normal distribution. This

implies that the dispersion of the residuals has a random nature and thereby lack of structural component that could considerably deform distribution of predicted values. Small deformation in the distribution of the predicted (Fig. 2c) with respect to observed (Fig. 2b)  $\lambda$  values confirms the above statement.

Taking into consideration not satisfactory prediction capability of Eq. (5) and the literature results [23,26] indicating that the thermal conductivity is more strongly correlated with air-filled porosity than with volume fractions of water or solids we used the air-filled porosity instead of water content in the regression Eqs. (7)–(10) using the  $B$  regression coefficients (Table 2) as for the equations with water content (Eqs. (3)–(6)). The air-filled porosity used was taken as the difference of saturated and current volumetric water contents.

Comparison of Figs. 2a and 3a indicate that the substitution of water content by air-filled porosity resulted in better agreement and considerably lower dispersion between observed and predicted thermal conductivities. Using penetration resistance and air-filled porosity in Eq. (9), selected for more detailed description, resulted in 50% improvement in accuracy as indicated by the RMSE values over using penetration resistance and water content in Eq. (5) (Table 3). At the same time MRE decreased from 60% to 34.2% and  $R^2$  increased from 0.768 to 0.942. Moreover, the distribution of predicted  $\lambda$  and residuals agree well with the theoretical normal distribution (Fig. 3b and c). Values of beta coefficients (Table 2) indicate that the relative contribution in the prediction of the thermal conductivity is greater for air-filled porosity ( $-0.99$ ) in Eq. (9) than water ( $0.926$ ) in Eq. (5). At the same time, the relative contribution of penetration resistance is greater in the equation with water

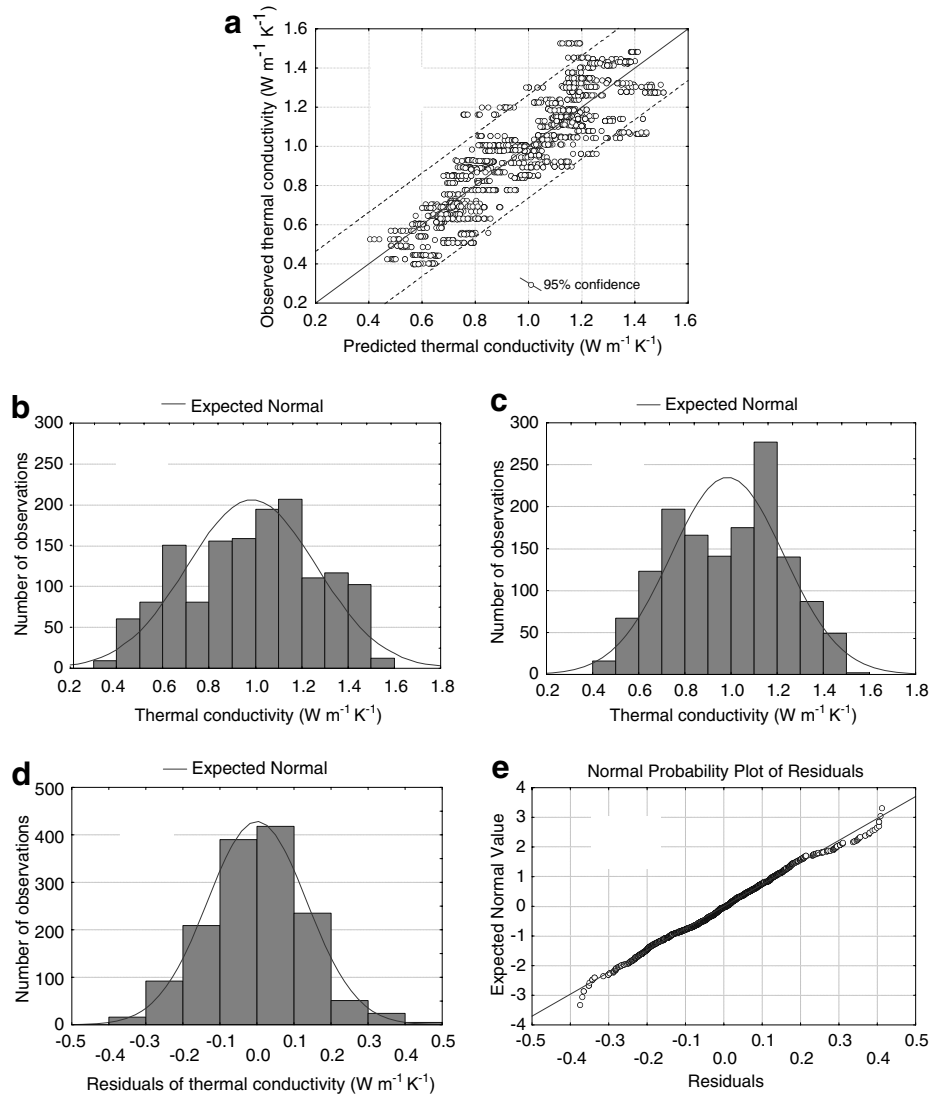


Fig. 2. Observed and predicted thermal conductivity by regression equation based on soil water content and penetration resistance with 95% confidence (a), histograms of  $\lambda$  observed (b), predicted (c), histogram of residuals with the theoretical normal distribution (solid line) (d), and normal probability plot of calculated (circles) and theoretical (solid line) residuals (e). The number of pairs of data is 1440.

(0.193) (Eq. (5)) than with air-filled porosity (0.114) (Eq. (9)). The negative sign of *beta* indicates negative relationship between the thermal conductivity and air-filled porosity.

Comparison of Figs. 2d and 3c indicate that the substitution of water content by air-filled porosity resulted in lower dispersion of residuals varying from  $-0.219$  to  $+0.179$  (Table 4). Distributions of the residuals and theoretical normal distribution (Fig. 3c and d) as well as nearly the same mean and median values (Table 4) indicate good agreement between both distributions. These indicate a random nature of the dispersion of the residuals and thereby lack of structural component. Moreover, deformation of the distribution of the predicted values (Fig. 3b) as compared to the observed (Fig. 2b) was lower for the equation with air-filled porosity (Eq. (9)) than with water (Eq. (5)).

Overall, the root mean square error data RMSE (Table 3) indicate that regression equations employing air-filled

porosity provide twice less dispersion of predictions for the thermal conductivity (RMSE 0.067–0.068) than those with volumetric water content (RMSE 0.134–0.135). These results agree with the findings of Ochsner et al. [23] indicating that the thermal conductivity is more strongly correlated with air-filled porosity than with volume fractions of water or solids.

Adding the content of sand in Eqs. (4), (6), (8) and (10) results in a slight improvement of predictions as compared with using just soil penetration resistance and water content or soil penetration resistance and air-filled porosity in Eqs. (3), (5), (7) and (9) (Table 3). This improvement was somewhat greater with respect to equations with PR and air-filled porosity (Eq. (10)) than PR and  $\theta$  (Eq. (6)). The statistical parameters of the equations were somewhat better with than without transformation of penetration resistance values into root squares (Eqs. (6), (10) vs. (4), (8)).

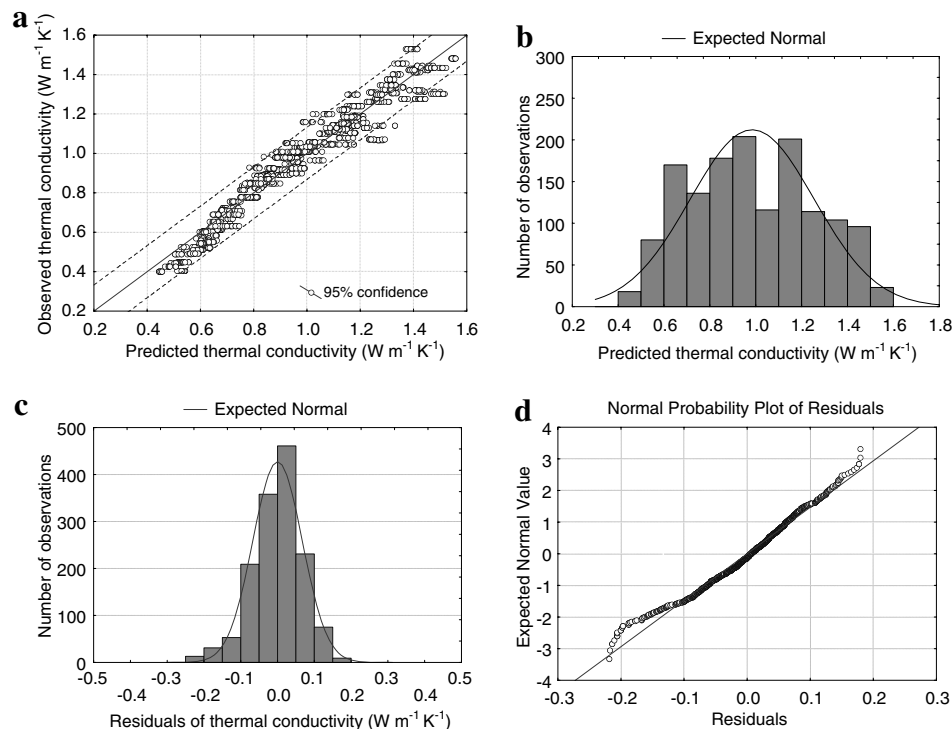


Fig. 3. Observed and predicted thermal conductivity by regression equation based on air-filled porosity and penetration resistance with 95% confidence (a), histograms of  $\lambda$  predicted (b), histogram of residuals with the theoretical normal distribution (solid line) (c), and normal probability plot of calculated (circles) and theoretical (solid line) residuals (d). The number of pairs of data is 1440.

Relative contribution of the sand in the predictions, as shown by  $\beta$  values (Table 2), was substantially smaller (0.043–0.053) than that of penetration resistance (0.113–0.223), water content (0.924–0.935) and air-filled porosity (–0.991 to –1.002). This relatively small effect of sand as a predictor of the  $\lambda$  can be due to relatively low differentiation of sand content (298.9–347.4 g kg<sup>–1</sup>) in our experimental plots. However, this effect can be much greater in areas with a greater variability of sand content where the regression equations can be useful in predicting spatial variability of  $\lambda$ . It is worthy to note that relative contribution of PR,  $\theta$  and air-filled porosity in prediction of the thermal conductivity was greater for the equations with than without content of sand (e.g. Eq. (6) vs. (5) and Eq. (10) vs. (9)).

All the modifications improving predictive accuracy, that is the substitution of water content by air-filled porosity, expressing penetration resistance in root squares and adding sand content, were included in Eq. (10) and resulted in the best statistical parameters with RMSE and MRE being 0.066 W m<sup>–1</sup> K<sup>–1</sup> and 32.3% and  $R^2$  of 0.944 (Table 3).

Our results support new developments of penetrometers equipped with TDR probe sensors for measuring volumetric soil water content [22,36,37,39] and thermo-time domain reflectometry probe for measuring soil thermal properties and water content [24]. The main benefits of combined measurements are that they are performed within the same volume and spatial location and thus prevent complications due to soil heterogeneity and moreover reduce measurement time and soil disturbance. Further

developments combining penetrometer probes with TDR and thermal sensors could allow additional improvement of the equations derived in our study. The developments for combined measurements are particularly useful in space missions where not only energy, but also volume and mass of the equipment are limited [21].

Furthermore, penetrometer data used for predicting the thermal conductivity in our study provide additional information on soil penetration resistance that is a useful measure of soil impedance to soil structure [20], trafficability and draft requirements [6] and root growth [13]. This makes the proposed approach convenient in several applications.

#### 4. Summary and conclusions

We have compared the thermal conductivities of the silt loam predicted by regression equations developed in this study based on easily measured values of penetration resistance and water content or air-filled porosity with those estimated by the statistical-physical model as characterized by a good estimation capability. Using penetration resistance and air-filled porosity in the regression equations resulted in a substantial improvement in accuracy over using penetration resistance and water content. Therefore, the equations based on penetration resistance and air-filled porosity are recommended for satisfactory predicting the thermal conductivity of soil. The accuracy of the equations can be further improved by adding sand content and transformation

of penetration resistance values to root squares. The relative contribution in the prediction of the thermal conductivity is greater for air-filled porosity than penetration resistance and sand content. There is a need for the use of integrated systems for combined measurements of penetration resistance and volumetric water content and/or air-filled porosity for further improvement of predictability and applicability of the equations by lessening spatial variability effects, reducing measurement time and disturbance of soil.

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